CORREZIONE DEL PARZIALE

$2z - 2|z|^2 + 6i\bar{z} = 7i$

$2(x^2 - y^2 + 2ixy) - 2(x^2 + y^2) + 6i(x - iy)0 = 7i$

$2x^2 - 2y^2 + 4ixy - 2x^2 - 2y^2 + 6ix + 6y = 7i$

$-4y^2 + 6y + 4ixy + 6ix = 7i$

$$\begin{cases} -4y^2 + 6y = 0 \\ 4xy + 6x = 7i \end{cases}$$

…da finire…

Le soluzioni, $z \in C \begin{cases} z^5 = -11i \\ \mathrm{Re}(z) \cdot \mathrm{Im}(z) \leq 0 \end{cases}$

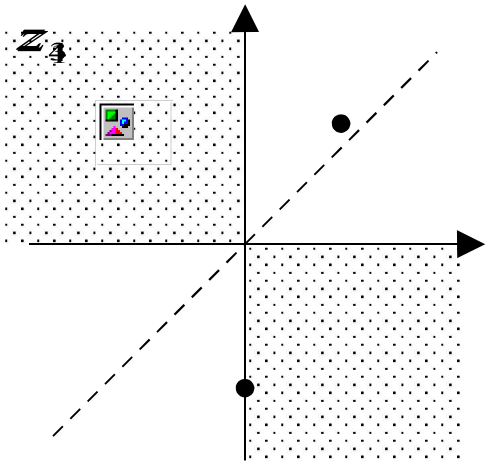
$z_0 = \sqrt[5]{11}e^{i+\frac{-\frac{\pi}{2}}{5}=i-\frac{\pi}{10}}$ IV quadrante

$z_1 = \sqrt[5]{11}e^{i+\frac{-\frac{\pi}{2}+2\pi}{5}} = \sqrt[5]{11}e^{i-\frac{3}{10}\pi}$ I quadrante

$z_2 = \sqrt[5]{11}e^{i+\frac{-\frac{\pi}{2}+4\pi}{5}} = \sqrt[5]{11}e^{i-\frac{7}{10}\pi}$ II quadrante

$z_3 = \sqrt[5]{11}e^{i-\frac{\frac{\pi}{2}+6\pi}{5}} = \sqrt[5]{11}e^{i-\frac{11}{10}\pi}$ III quadrante

$z_4 = \sqrt[5]{11}e^{i-\frac{\frac{\pi}{2}+8\pi}{5}} = \sqrt[5]{11}e^{i-\frac{15}{10}\pi=\frac{3}{2}\pi}$ IV quadrante



Le soluzioni sono 3.

$y(x) = \frac{\log(x^8)}{8} - \frac{4}{x+8} \quad x \neq -8 \quad x \neq 0 \quad D = \Re \setminus \{-8,0\}$

$f'(x) = \left(\frac{1}{8}8x^7 \cdot \frac{1}{x^8}\right) + \frac{4}{(x+8)^2} = \frac{1}{x} + \frac{4}{(x+8)^2} = \frac{(x+8)^2 + 4x}{x(x+8)^2}$

$D > 0$

$x^2 + 64 + 20x = 0$

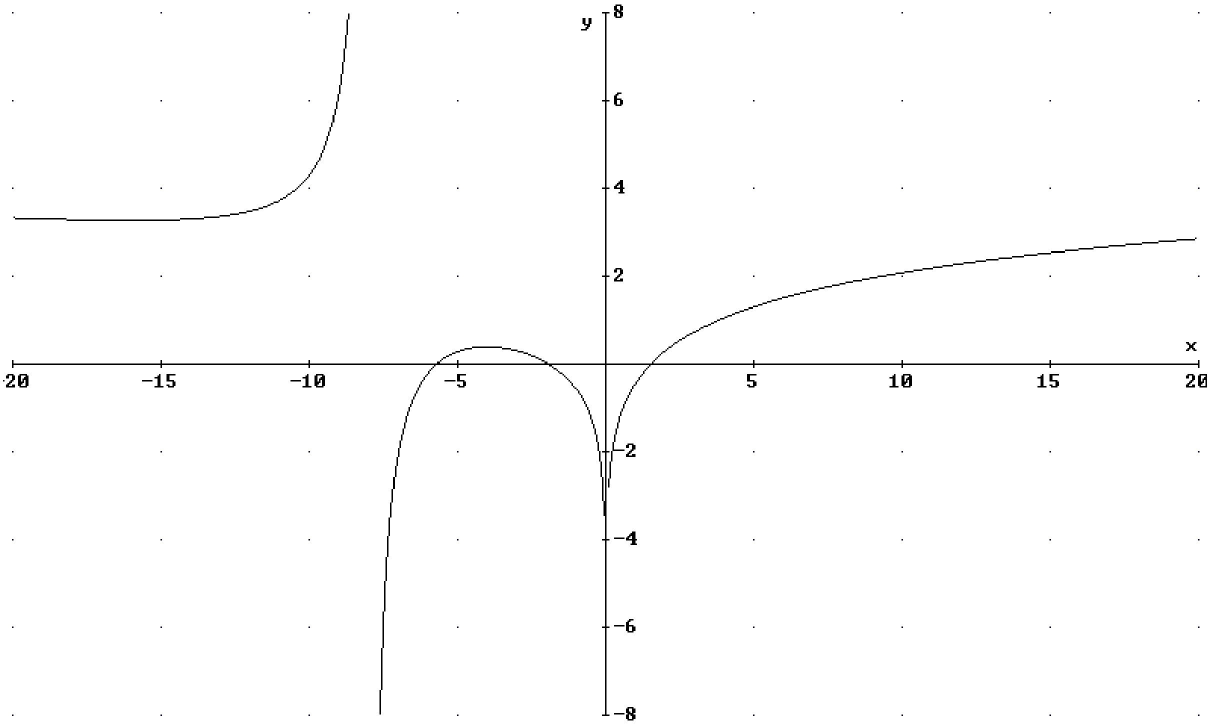
$x = -10 \pm \sqrt{100 - 64} = -10 \pm 6 = \begin{vmatrix} -16 \\ -4 \end{vmatrix}$

-	-	- 0	+
-16+	-4	+	+
-	+	-	+

$\lim_{x\to+\infty} \frac{\log(x^8)}{8} - \frac{4}{x+8} = +\infty \quad \lim_{x\to-8^+} \frac{\log(x^8)}{8} - \frac{4}{x+8} = +\infty$

$\lim_{x\to-\infty} \frac{\log(x^8)}{8} - \frac{4}{x+8} = -\infty \quad \lim_{x\to-8^-} \frac{\log(x^8)}{8} - \frac{4}{x+8} = -\infty$

$\lim_{x\to0^+} \frac{\log(x^8)}{8} - \frac{4}{x+8} = -\infty \quad \lim_{x\to0^-} \frac{\log(x^8)}{8} - \frac{4}{x+8} = -\infty$



$y(A) = \Re$

$y(x) = 20$ 3 soluzioni

$f(x) = 2\log(|x+1|) - x^2 + 2x$

Quanti punti critici ha?

$x > -1$

$f(x) = 2\log(x+1) - x^2 + 2x$

$f'(x) = 0 \Leftrightarrow \frac{2}{x+1} - 2x + 2 = 0$

$\frac{1}{x+1} - x + 1 = 0$

$\frac{1 - x(x+1) + x + 1}{x+1} = 0$

$-\sqrt{2} < x < \sqrt{2}$

$x < -1$

$f_2(x) = 2\log(-x+1) - x^2 + 2x$

$f_2(x) = \frac{2(-1)}{-x-1} - 2x + 2 = 0$

$\frac{-1}{-(x+1)} - x + 1 = 0$

$\frac{-1 - x(-x-1) + 1}{-x-1} = 0$

$-\sqrt{2} < x < \sqrt{2}$

Abbiamo scartato alcuni valori perché dobbiamo prendere solo quelli per $x > -1$ e $x < -1$.
Ci sono 2 punti critici (due punti di massimo) in $-\sqrt{2}$ e $\sqrt{2}$.

POLINOMIO DI TAYLOR

1. $f(x) = g(x) + d(f(x)) \Leftrightarrow f(x) \approx g(x), (x \to x_0) \Leftrightarrow \lim_{x \to x_0} \frac{f(x)}{g(x)} = 1$

2. $f(x) \underset{x \to x_0}{\approx} g(x) \Rightarrow \lim_{x \to x_0} f(x) = \lim_{x \to x_0} g(x)$

3. $o(g) + o(g) = o(g)$

   $c \cdot o(g) = o(g)$

   $f \cdot o(g) = o(f \cdot g)$

   $o(-g) = o(g)$

$$L = \lim_{x \to 0} \frac{\sqrt{1 + x^2} - \exp(x^2) + sen\left(\frac{x^2}{2}\right)}{tg\left(x^4\right)}$$

$$f = \sum_{k=0}^{n} f^{(k)}(x_0) \frac{\left(x - x_0\right)}{k!} + o\left(x - x_0\right)^k$$

Sviluppo in x_0

$$\exp(x) = f^{(0)}(0) + f'(0)x + \frac{f''(0)(x)}{2!} + \frac{f'''(0)x}{3!} + o(x^3)$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^3)$$

$$sen(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2} + f'''(0)\frac{x^3}{6} + o(x^3)$$

$$sen(x) = \cos(0)x - \cos(0)\frac{x^3}{6} = x - \frac{x^3}{6} + o(x^3)$$

$$\cos(x) = 1 + \frac{x^2}{2} + o(x^3)$$

$$tg(x) = x + \frac{x^3}{6} + o(x^3)$$

$$senh(x) = x + \frac{x^3}{3!} + o(x^3)$$

$$\cosh(x) = 1 + \frac{x^2}{2} + o(x^3)$$

$$\left(1 + x\right)^{\alpha} = \sum_{k=0}^{n} \binom{\alpha}{k} x^k + o(x^4)$$

$\binom{\alpha}{k} = \frac{\alpha!}{k!\left(\alpha - k\right)!}$ coefficiente binomiale

$$\left(1+x^2\right)^{\frac{1}{2}} = 1+\frac{x^2}{2}-\frac{x^4}{8}+o(x^4)$$

$$e^{x^2} = 1+x^2+\frac{x^4}{2}+o(x^4)$$

$$sen\left(\frac{x^2}{2}\right)=\frac{x^2}{2}+o(x^2)$$

$$tg(x)=x^4+o(x^4)$$

$$L=\frac{\cancel{1}+\frac{\cancel{x^2}}{\cancel{2}}+\frac{x^4}{8}+o(x^4)-\left(\cancel{1}+\cancel{x^2}+\frac{x^4}{2}+o(x^4)\right)+\frac{\cancel{x^2}}{\cancel{2}}+o(x^4)-\frac{x^4}{8}-\frac{x^4}{2}+o(x^4)}{x^4+o(x^4)}\approx\frac{-\frac{x^4}{8}-\frac{x^4}{2}}{x^4}=$$

$$=\frac{\frac{-x^4-4x^4}{8}}{x^4}=-\frac{5}{8}$$

$$L=\lim_{x\to 0}\frac{e^{x^2}-\cosh\left(x\sqrt{2}\right)}{\left(x-sen(x)\right)tg(x)}$$

$$\cosh(y)=1+\frac{y^2}{2}+\frac{y^4}{24}+o(y^4)\Rightarrow\cosh\left(x\sqrt{2}\right)=1+\frac{2x^2}{2}+\frac{4x^4}{24}+o(x^4)$$

DEN:

$$sen(x)=x-\frac{x^3}{6}+o(x^3)$$

$$tg(x)=x+o(x)$$

$$\left(x-x+\frac{x^3}{6}+o(x^3)\right)x=\left(\frac{x^3}{6}+o(x^3)\right)x=\frac{x^4}{6}+o(x^4)$$

NUM:

$$1+x^2+\frac{x^4}{2}+o(x^4)-1-x^2-\frac{x^4}{6}+o(x^4)=\frac{x^4}{2}-\frac{x^4}{6}+o(x^4)$$

$$L=\frac{\frac{x^4}{2}-\frac{x^4}{6}+o(x^4)}{\frac{x^4}{6}+o(x^4)}\approx\frac{\frac{x^4}{2}-\frac{x^4}{6}}{\frac{x^4}{6}}=2$$

$$I_a = \lim_{x \to 0} \frac{x^{3x} - 1 - senh(x^a)}{x^a}, a > 0$$

$$x^{3x} = e^{3x\log(x)}$$

$$e^x = 1 + x + o(x)$$

$$e^{3x\log(x)} = 1 + 3x\log(x) + o(x\log(x))$$

$$senh(x^a) = x^a + o(x^a)$$

$$= \frac{\cancel{1} + 3x\log(x) - \cancel{1} - x^a + o(x\log(x)) + o(x^a)}{x^a}$$

Per calcolare chi va a 0 più lentamente:

$$\lim_{x \to 0} \frac{x\log(x)}{x^a} = \lim_{x \to 0} \frac{\log(x)}{x^{a-1}} = \begin{cases} 0, a < 1 \Rightarrow x\log(x) = o(x^a) \\ -\infty, a \geq 1 \Rightarrow x^a = o(x\log(x)) \end{cases}$$

$a \geq 1$

$$x^a = o(x\log(x))$$

$$I_a = \frac{3x\log(x) + o(x\log(x))}{x^a}$$

$$I_a = -\infty$$

$a < 1$

$$I_a \approx \lim = \frac{-x^a}{x^a} = -1$$

A. I_a non soddisfa B,C,D;

B. I_a esiste finito $\forall a > 0$;

C. $I_a = -\infty$ se e solo $a = 1$;

D. $I_a < 0, \forall a > 0$. VERA

L'INTEGRALE GENERALIZZATO

$I = \int_0^3 \frac{\sqrt{1+t} - 1}{t} dt$ vale…

sostituiamo $1 + t = x^2, x \geq 0$

$$\Rightarrow t = x^2 - 1$$

$$\Rightarrow dt = 2xdx$$

$$t = 3; x^2 = 4 \Rightarrow x = 2$$

$$t = 0; x^2 = 1 \Rightarrow x = 1$$

$$\int_1^2 \frac{x-1}{x^2-1} 2xdx = \int_1^2 \frac{2x(x-1)}{(x+1)(x-1)} dx = 2\int_1^2 \frac{2x}{x+1} dx = 2\int_1^2 \frac{x+1-1}{x+1} dx = 2\int_1^2 \left(1 - \frac{1}{x+1}\right) dx = 2\int_1^2 dx - 2\int_1^2 \frac{dx}{x+1} =$$

$$= 2\left(x\Big|_1^2 - \log(x+1)\Big|_1^2 \right) = 2(1 - \log(3) + \log(2)) = 2 - 2\log\left(\frac{3}{2}\right) = 2 - \log\left(\frac{9}{4}\right)$$

Osservazione: $\int \frac{dx}{x^a} = \log|x + a|$

L'integrale converge?

$c \in \Re \quad \int_0^c \frac{dx}{x^\beta} < +\infty \Leftrightarrow \beta < 1$ (cioè converge)

$\int_c^{+\infty} \frac{dx}{x^\beta} < +\infty \Leftrightarrow \beta > 1$

$f \to 0$

$\sqrt{1+t} = 1 + \frac{1}{2}t + o(t)$

$\frac{\sqrt{1+t} - 1}{t} \approx \frac{1 + \frac{t}{2} - 1}{t} \approx \frac{1}{2}x^0$

esponente $\beta = 0$

$f(x) = \frac{x}{x^2 - 9}$

$\frac{A}{x+3} + \frac{B}{x-3} = \frac{x}{(x+3)(x-3)}$

$Ax - 3A + Bx + 3x = x$

$x(A+B) - 3(A-B) = x \begin{cases} A + B = 1 \\ A - B = 0 \end{cases}$

$A = B = \frac{1}{2}$

$f(x) = \frac{\frac{1}{2}}{x-3} + \frac{\frac{1}{2}}{x+3}$

1) $F(t) = \int_0^t f(x)dx$ ha dominio *I*=

2) calcolare *F(t)*

3) $F(t) = \log\left(\frac{2}{3}\right) \Leftrightarrow t = ?$

4) Calcolare *F(I)*

$F(t) = \int_0^t f(x)$

$f(x) = \frac{x}{x^2 - 9}$

$D(f) =]-\infty;-3[\cup]-3:3[\cup]3;+\infty[$

1) *F(t)* è definita in $]-3;3[$

$F(t) = \frac{1}{2}\int_0^t \frac{2x}{x^2 - 9}dx = \frac{1}{2}\left(\log\left|x^2 - 9\right| \Big|_0^t \right) = \frac{1}{2}\log\left(9 - x^2\right)\Big|_0^t = \frac{1}{2}\left(\log\left(9 - t^2\right) - \log(9)\right)$

2) $\frac{1}{2}\left(\log\left(9 - t^2\right) - \log(9)\right) = \log\left(\frac{2}{3}\right)$

$$\frac{1}{2}\log\left(9-t^2\right)-\frac{1}{2}\log(9)=\log(2)-\log(3)$$

$$\frac{1}{2}\log\left(9-t^2\right)-\frac{1}{2}\log(9)=\log(2)-\log(3)$$

$$\frac{1}{2}\log\left(9-t^2\right)-\log(3)=\log(2)-\log(3)$$

$$\frac{1}{2}\log\left(9-t^2\right)=\log(2)$$

$$\log\left(9-t^2\right)=\log(4)$$

$$9-t^2=4$$

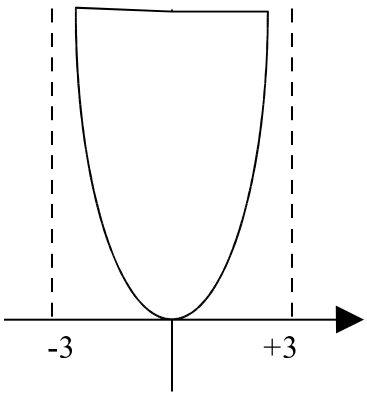
$$t^2=5$$

$$t=\pm\sqrt{5}$$

$$I=\left]-3;+3\right[$$

$$F(I)=?$$

Studio dell'andamento di *F'(x)=f(x)*



$$F(I)=\left[0;+\infty\right[$$

L'INTEGRALE GENERALIZZATO

$$I_a = \int_0^{+\infty} \frac{(x-1)|sen(x)|}{x^a(x^2+1)} dx$$

Studiare la convergenza di I_a al variare di a in $x \to 0$

$x - 1 \approx -1$

$sen(x) \approx x$

$x^2 + 1 \approx 1$

$$\frac{(x-1)|sen(x)|}{x^a(x^2+1)} \approx \frac{-1|x|}{x^a} \approx \frac{-1}{x^{a-1}}$$

se $a - 1 < 1 \Rightarrow a < 2$

$x \to +\infty$ $\frac{(x-1)|sen(x)|}{x^a(x^2+1)} \approx \frac{x}{x^{a+2}} = \frac{1}{x^{a+1}}$

per $a > 0$ converge.

I_a converge $\forall a, 0 < a < 2$

$I_a = \int_0^{+\infty} \frac{arctg(x^a)}{x^a + x} dx$ Studiare la convergenza di I_a al variare di a.

$arctg(x^a) \approx x^a$

$x^a + 1 \approx 1, (a \geq 0)$

in $x \to 0$

$$\frac{x^a}{x} = x^{a-1} = \frac{1}{x^{1-a}}$$

$1 - a < 1$

$-a < 0$

$a > 0$

in $x \to +\infty$

$$arctg(x^2) \approx \frac{\pi}{2} \approx \frac{\pi}{2x^{a+1}}$$

$(x^a + 1)x \approx x^{a+1}$

I_a converge $\Leftrightarrow$ $\begin{matrix} a+1>1 \\ a>0 \end{matrix}$

$$I = \int_1^2 \frac{dx}{x(x^2+1)}$$

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$A(x^2+1) + Bx^2 + Cx = 1$

$(A+B)x^2 + Cx + A = 0$

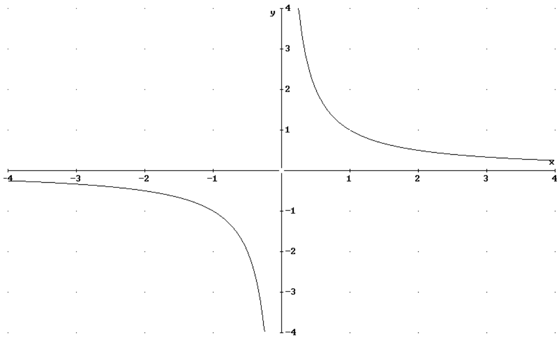
$A = 1; B = -1$

$$\int_1^2 \frac{1}{x}dx - \frac{1}{2}\int_1^2 \frac{2x}{x^2+1} = \log|x|\Big|_{x=1}^{x=2} - \frac{1}{2}\log\left(x^2+1\right)\Big|_{x=1}^{x=2} = \log(2) - \log(1) - \frac{1}{2}\log(5) + \frac{1}{2}\log(2) =$$

$$= \frac{3}{2}\log(2) - \frac{1}{2}\log(5) = \log\sqrt{\frac{8}{5}}$$

La convergenza dipende dalla continuità e dalla limitatezza della funzione.

Non è integrabile per $x \to 0$



$\frac{1}{x}$

$$I_a = \int_0^{+\infty} \frac{arctg(x)}{x^a(x+1)}dx$$

$(x \to 0)$

$arctg(x) \approx x + o(x)$

$x + 1 \approx 1$

$$f(x) \approx \frac{x}{x^a} = \frac{1}{x^{a-1}}$$

$$\int_0^c x^\beta dx \text{ converge} \Leftrightarrow \begin{matrix} \beta > -1 \\ a-1 > 0; a > 0 \end{matrix}$$

$$f(x) \approx \frac{\pi}{2}\frac{1}{x^{a+1}}$$

$$\int_c^{+\infty} x^\beta dx \text{ converge} \Leftrightarrow \beta < -1$$

$1 - a < -1 \Rightarrow -a < -2 \Rightarrow a > 2$

$x \to 0 \quad f(x) \approx x^{1-a} \quad 1 - a > -1 \Rightarrow -a > -2 \Rightarrow a < 2$

$x \to +\infty \quad f(x) \approx \frac{\pi}{2}\frac{1}{x^{a+1}} = x^{-a-1} \quad -a - 1 < -1 \Rightarrow -a < 0 \Rightarrow a > 0$

$0 < a < 2$

$$I_{a,b} = \int_0^{+\infty} \frac{\left(x^a - 1\right)arctg(x)}{\left(x^\beta + 1\right)}dx \quad a, b > 0$$

$x \to \infty$	$x \to 0$
$x^a - 1 \approx x^a$	$x^a - 1 \approx -1$
$x^\beta + 1 \approx x^\beta$	$x^\beta + 1 \approx +1$
$arctg(x) \approx \frac{\pi}{2}$	$arctg(x) \approx x$

$$f = \frac{x^a \pi}{2x^\beta} \qquad f = x^{a-b} \qquad a - b < -1 \qquad a < b - 1$$